Team (5 problems)

1. Show that a closed simply connected 3-manifold is homotopically equivalent to the 3-sphere.

2. Recall that a symplectic form ω on a smooth manifold M is a degree 2 differential form which is closed and non-degenerate. Here non-degeneracy means that if there is a point $x \in M$ and a tangent vector $u \in T_xM$ such that $\omega(u, v) = 0$ for any $v \in T_xM$, then $u = 0$. Let ω be a symplectic form on M.

a). Show that M is orientable.

b). A vector field V on M is called a Liouville vector field with respect to ω if $L_V\omega = \omega$. Here L_V denotes the Lie derivative wit respect to V. Show that there isn't any Liouville vector field on M if M is a closed manifold.

3. Prove that there is no nonconstant continuous function on R which is periodic with respect to two periods 1 and π.

4. If M is a compact manifold with negative sectional curvature, then the isometry group of M is finite.

5. (E. Cartan) Let K be a compact Lie group acting by isometries on a simply connected, complete Riemannian manifold M of negative curvature. Then there is a common fixed point of all $k \in K$.